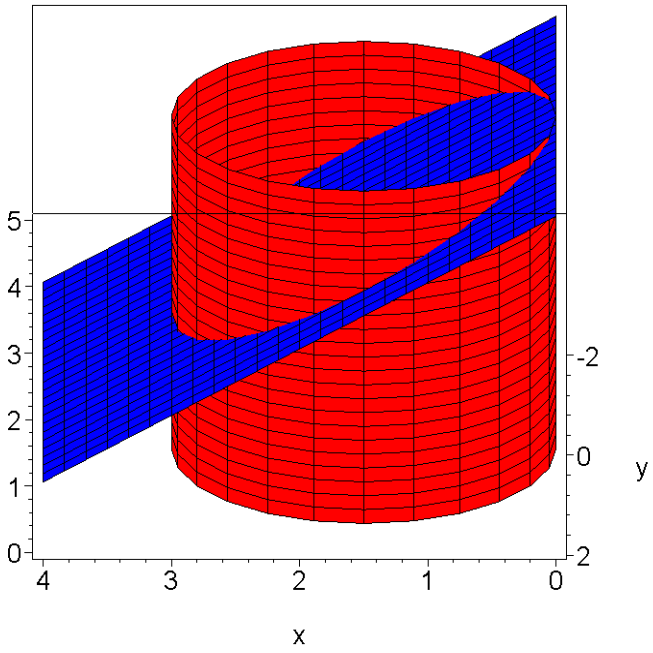
```
> restart:with(plots):with(student):
Warning, the name changecoords has been redefined

> # problem 1


> pl1:=plot3d( 3*cos(theta), theta=0..Pi,z=0 ..
  5,coords=cylindrical,color = red, axes=boxed):
> pl2:=plot3d( 5-x, x = 0 .. 4,  y= -2  .. 2,color = blue,
  axes=boxed):
>

> display3d(pl1,pl2);# you have to twist it to see it
```

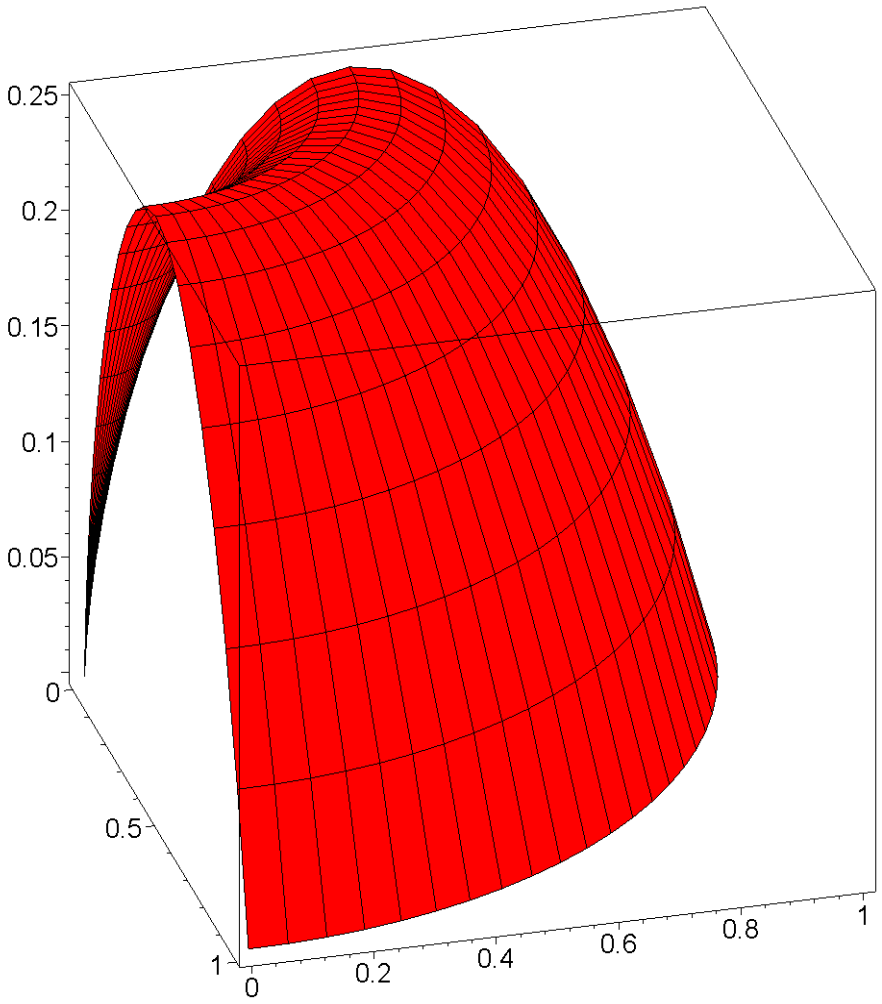



```
> Int(Int(Int(r, z= 0 .. 5-r*cos(theta) ),r=0 .. 3*cos(theta)),
  theta=0 ..Pi)=int(int(int(r, z= 0 .. 5-r*cos(theta) ),r=0 ..
  3*cos(theta)), theta=0 ..Pi);
```

$$\int_0^{\pi}\int_0^{3\cos(\theta)}\int_0^{5-r\cos(\theta)} r\,dz\,dr\,d\theta = \frac{63\,\pi}{8}$$

```
> # Ex 2
> #
> plot3d( 1-cos(phi),theta=0..Pi/2,phi = 0..
  Pi/2,coords=spherical,color = red, axes=boxed);
> Int(Int(Int(rho^2*sin(phi), rho= 0 .. 1-cos(phi) ),phi=0 .. Pi/2),
```

```
theta=0 ..Pi/2)=int(int(int(rho^2*sin(phi), rho= 0 .. 1-cos(phi)
),phi=0 .. Pi/2), theta=0 ..Pi/2);
```

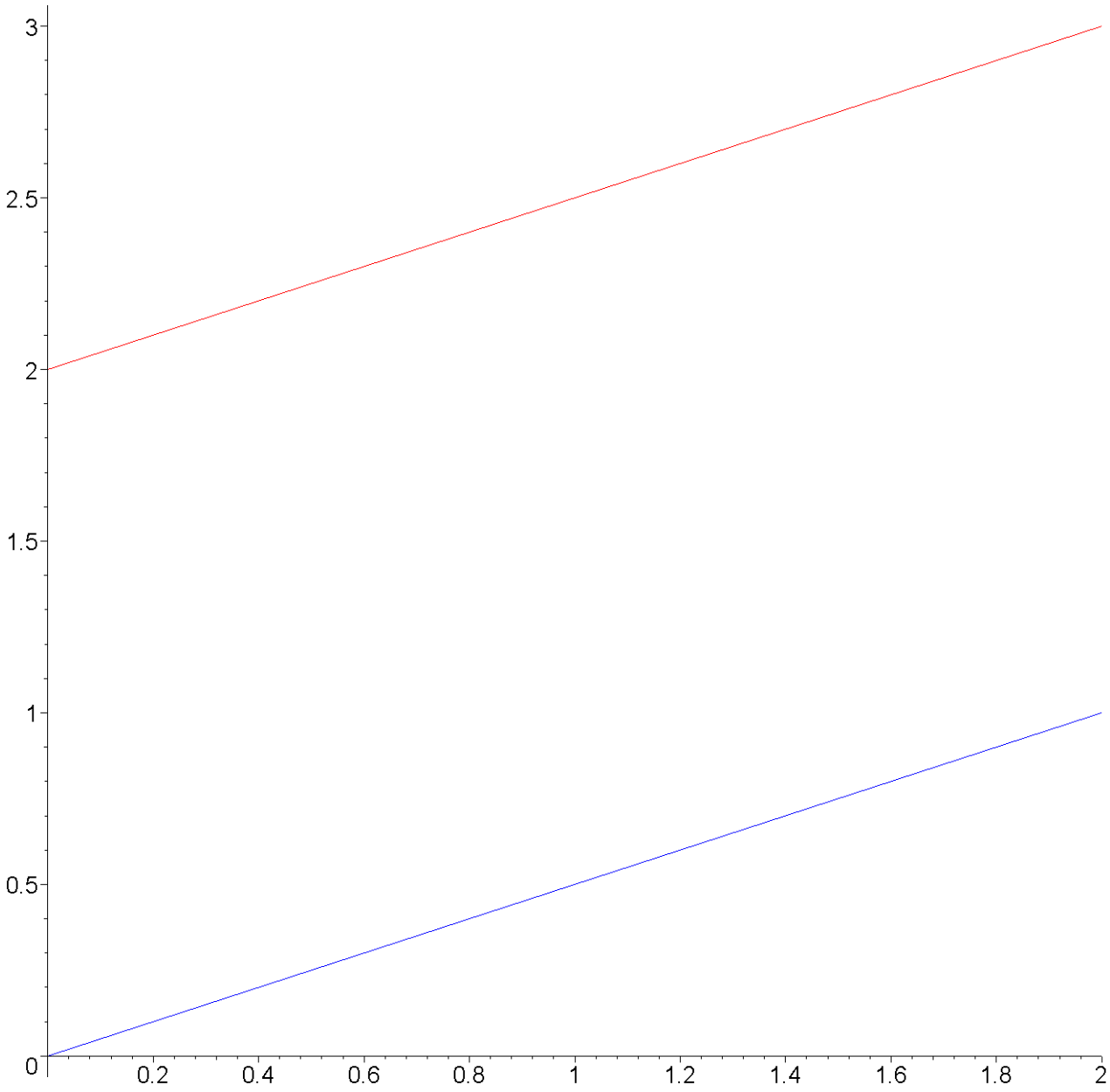


$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{1-\cos(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{\pi}{24}$$

```
> # Ex3
> P1:=plot( (x +4)/2,  x= 0 ..2,color=red):P2:=plot( x/2,  x= 0
  ..2,color=blue):
> with(VectorCalculus);
Warning, the assigned names <,> and <|> now have a global binding
```

```
Warning, these protected names have been redefined and unprotected: *, +, -, ., D,
Vector, diff, int, limit, series

[&x, *, +, -, ., <,>, <|>, AddCoordinates, ArcLength, BasisFormat, Binormal, CrossProd,
    CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProd, DotProduct, Flux,
    GetCoordinateParameters, GetCoordinates, Gradient, Hessian, Jacobian, Laplacian, LineInt,
    MapToBasis, Nabla, Norm, Normalize, PathInt, PrincipalNormal, RadiusOfCurvature,
    ScalarPotential, SetCoordinateParameters, SetCoordinates, SurfaceInt, TNBFrame, Tangent,
    TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential,
    Wronskian, diff, evalVF, int, limit, series]
> display(P1,P2);
```




```
> Int(Int(x^3*(2*y-x)*exp((2*y-x)^2), y= x/2 .. (x+4)/2),x=0 ..
  2)=int(int(x^3*(2*y-x)*exp((2*y-x)^2), y= x/2 .. (x+4)/2),x=0 ..
  2);
>
```

$$\int_0^2 \int_{\frac{x}{2}}^{\frac{x}{2}+2} x^3 (2y-x)\, \mathbf{e}^{((2y-x)^2)}\, dy\, dx = -1 + \mathbf{e}^{16}$$

```
> # change in var is u=2*y-x and v=x which gives the following
  region of integration then x=v and y= (u+v)/2
```

```
> # top limit of y=(x+4)/2 becomes u=4
> # bottom limit of y=x/2 becpmes u=0
> # x =2 becomes v=2 and x=0 becomes v=0
>
> # compute the Jacobian
```

Change coordinate systems.

```
> (M,d):=Jacobian( [v,(u+v)/2], [u,v], 'determinant' );
```

$$M, d := \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \frac{-1}{2}$$

```
> # jacobian is 1/2
```

```
> .5*Int(Int(v^3*u*exp((u)^2), v=0 ..2),u=0 .. 4)=
  (.5)*int(int(v^3*(u)*exp((u)^2), u=0..4),v=0 .. 2);
```

$$-0.5 \int_0^4 \int_0^2 v^3\, u\, \mathbf{e}^{(u^2)}\, dv\, du = 1.0 - 1.0\, \mathbf{e}^{16}$$

```
> # ex4
> Int(Int(ln(y/x), x= sqrt(y) .. y),y=1 .. 4)=int(int(ln(y/x), x=
  sqrt(y) .. y),y=1 .. 4);
>
```

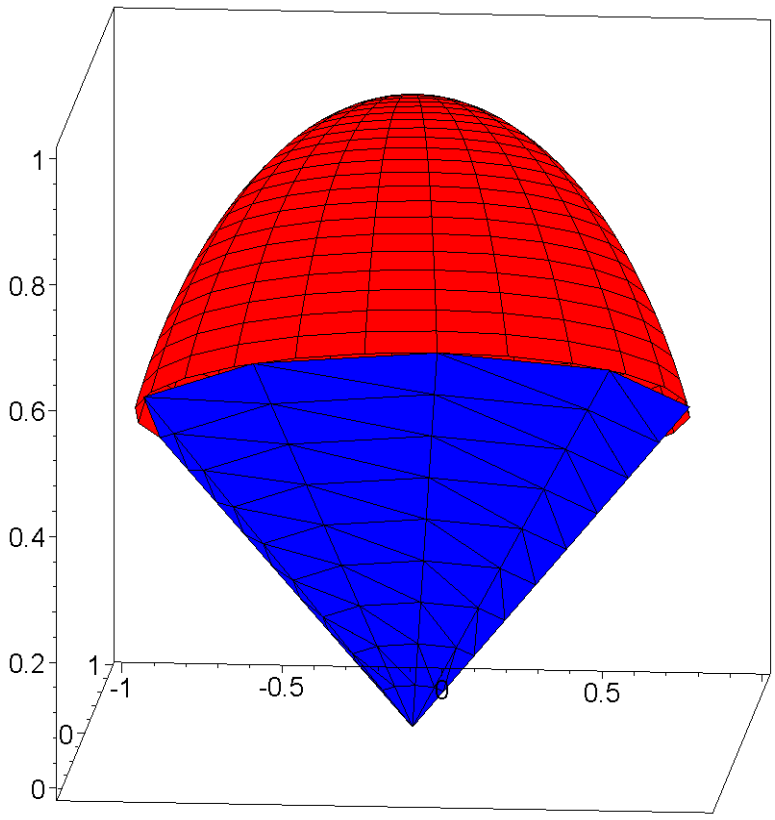
$$\int_1^4 \int_{\sqrt{y}}^{y} \ln\left(\frac{y}{x}\right) dx\, dy = \frac{79}{18} - \frac{16}{3} \ln(2)$$

```
> Int(Int(Int(rho^2*sin(phi)*cos(phi),rho = 0 .. cos(phi)), phi= 0
  .. Pi/2),theta=0 .. 2*Pi)=int(int(int(rho^2*sin(phi)*cos(phi),rho
  = 0 .. cos(phi)), phi= 0 .. Pi/2),theta=0 .. 2*Pi);
```

$$\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\cos(\phi)} \rho^2 \sin(\phi) \cos(\phi)\, d\rho\, d\phi\, d\theta = \frac{2\pi}{15}$$

```
> #ex5
> pl1:=plot3d( 1,theta=0..2*Pi,phi = 0.. Pi/3,coords=spherical,color
  = red, axes=boxed):
> pl2:=implicitplot3d( phi = Pi/3, rho = 0 .. 1,  theta = 0 ..Pi*2,
  phi = 0 .. Pi/2,coords=spherical,color = blue, axes=boxed):

> display3d(pl1,pl2);# you have to twist it to see it
```




```
> Int(Int(Int(rho^2*sin(phi),rho = 0 .. 1), phi= 0 .. Pi/3),theta=0
  .. 2*Pi)=int(int(int(rho^2*sin(phi),rho = 0 .. 1), phi= 0 ..
  Pi/3),theta=0 .. 2*Pi);
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{1}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{\pi}{3}$$

```
> #Ex 6
> PathInt(x^3/y, [x,y] = Path(<t,t^2/2>,t=0 .. 2 ),inert ) +
  PathInt(x^3/y, [x,y] = Line( <2,2>, <4,3> ),inert )=PathInt(x^3/y,
  [x,y] = Path(<t,t^2/2>,t=0 .. 2 ) ) + PathInt(x^3/y, [x,y] = Line(
```

```
  <2,2>, <4,3> ) );evalf(%);
```

$$\int_0^2 2\,t\sqrt{1+t^2}\,dt + \int_0^1 \frac{(2+2\,t)^3\sqrt{5}}{2+t}\,dt =$$

$$-\frac{-\frac{20\sqrt{\pi}\sqrt{5}}{3}+\frac{4\sqrt{\pi}}{3}}{2\sqrt{\pi}} + 8\sqrt{5}\,\ln(2) + \frac{44\sqrt{5}}{3} - 8\sqrt{5}\,\ln(3)$$

$$32.32937658 = 32.32937656$$

```
> #Ex 7 Read p. 1146
> delta:= (x,y,z) -> 2-z;
```

$$\delta := (x, y, z) \to \mathit{VectorCalculus}{:-}\grave{+}\grave{}(2, \mathit{VectorCalculus}{:-}\grave{-}\grave{}(z))$$

```
> PathInt((x^2+y^2)*(2-z), [x,y,z] = Path(<0,sin(t),cos(t)>,t=0 ..
  Pi ),inert ) =PathInt((x^2+y^2)*(2-z), [x,y,z] =
  Path(<0,sin(t),cos(t)>,t=0 .. Pi ) ) ;
```

$$\int_0^{\pi} \sin(t)^2\,(2-\cos(t))\sqrt{\cos(t)^2+\sin(t)^2}\,dt = \pi$$

```
> #Ex8
> SetCoordinates( 'cartesian'[x,y,z] );
> LineInt( VectorField( <y+z,x+z,x+y> ), Path( <t,t^2,t^4>, t=0..1
  ),inert )=LineInt( VectorField( <y+z,x+z,x+y> ), Path(
  <t,t^2,t^4>, t=0..1 ) );
```

$$\mathit{cartesian}_{x,\,y,\,z}$$

$$\int_0^1 t^2 + t^4 + 2\,(t+t^4)\,t + 4\,(t+t^2)\,t^3\,dt = 3$$

```
> #EX9
> SetCoordinates( 'cartesian'[x,y] );
> LineInt( VectorField( <-y,x> ), Path( <3*cos(t),3*sin(t)>, t=0..Pi
  ),inert )+ LineInt( VectorField( <-y,x> ), LineSegments( <-3,0>,
  <3,0>),inert )=LineInt( VectorField( <-y,x> ), Path(
  <3*cos(t),3*sin(t)>, t=0..Pi ) )+ LineInt( VectorField( <-y,x> ),
  LineSegments( <-3,0>, <3,0>) );
```

$$\mathit{cartesian}_{x,\,y}$$

$$\int_0^{\pi} 9\sin(t)^2 + 9\cos(t)^2\,dt + \int_0^1 0\,dt = 9\,\pi$$

```
> # for Flux we must use the Line Integral Form p.1157 (4)where M =
  -y and N = x Note need -N in integral
Error, (in VectorCalculus:-Vector[column]) not enough variable names were given
```

```
> LineInt( VectorField( <-y,-x> ), Path( <3*cos(t),3*sin(t)>,
  t=0..Pi ),inert )+ LineInt( VectorField( <-y,-x> ), LineSegments(
  <-3,0>, <3,0>),inert )=LineInt( VectorField( <-y,-x> ), Path(
  <3*cos(t),3*sin(t)>, t=0..Pi ) )+ LineInt( VectorField( <-y,-x> ),
  LineSegments( <-3,0>, <3,0>) );
```

$$\int_0^{\pi} 9\sin(t)^2 - 9\cos(t)^2\,dt + \int_0^{1} 0\,dt = 0$$

```
> #it was zero why? exact!!
>
>
```